STABILITY OF FORWARD AND REVERSE WET COMBUSTION FRONTS IN POROUS MEDIA

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ABSTRACT

A linear stability analysis shows that reverse wet combustion is unstable for nearly all physically realizable operating conditions for the special case of coincident steam and combustion fronts. Expansion effects due to gas generation from combustion and vaporization are found to have a stabilizing influence on reverse wet combustion.

Forward wet combustion is also found to be conditionally stable. Significant expansion effects at the combustion front can overwhelm the stabilizing influence of a favorable mobility ratio and effect destabilization with respect to oscillatory long wavelength modes.

INTRODUCTION

In situ combustion processes are being considered for a variety of recovery schemes for underground fossil fuel deposits. These include combustion processes for secondary recovery of highly viscous crude oils, tertiary recovery of lighter oils, in situ retorting of oil shale, oil recovery from tar sands, and in situ coal gasification. It is useful to categorize all in situ combustion processes as either forward or reverse combustion. In the former the combustion front travels in the same direction as the flow of gases, whereas in the latter it travels countercurrent to the direction of gas flow.

Forward combustion is by far the more widely used; in such applications as secondary and tertiary oil recovery, it is effectively the only combustion process used. However, reverse combustion offers particular advantages for in situ thermal recovery schemes for relatively impermeable media such as subbituminous coal, lignites, and tar sands. The reason for this is that during forward combustion, tars vaporized at the combustion front are convected into cooler regions ahead of the front where they condense and thus reduce the natural permeability of the bed. In contrast, for reverse combustion the vaporized tars, or other high molecular weight compounds generated by the combustion, travel toward the production well through a heated region whose permeability is usually greater than the natural permeability of the bed since the front has passed through it. In technologies such as the linked vertical well process for in situ coal gasification, a reverse combustion linking step is used to create a highly permeable link through which the combustion gases from subsequent forward gasification can escape. The use of reverse combustion as a preparatory step prior to forward combustion is also being considered for tar sands.

Forward and reverse in situ combustion may also be either a dry or wet combustion process. In the former only air is injected into a relatively dry fossil fuel deposit such that any vaporization of water due to conduction of heat from the combustion front, has no effect on the process. In a wet combustion process either water is injected along with air or there is sufficient water naturally present in the deposit, such that the steam front is in relative close proximity to the combustion front. If enough water is present, the steam front can be coincident with the combustion front and thus depress the combustion front temperature to the saturation temperature of steam at the prevailing pressure.

Forward wet combustion has some advantages for secondary recovery processes for highly viscous crude oils because less air is required for the front to traverse a given distance thus making it more economically attractive. The authors are particularly interested in the influence of wet combustion on the sweep efficiency of both the reverse and forward combustion steps for the in situ coal gasification process. This process is being considered for the gasification of western subbituminous coal seams which can contain considerable moisture.

The sweep efficiency of many in situ combustion processes is related to the stability of the process. In a stable combustion process any perturbations in the combustion front, due to heterogeneities in the properties of the porous media, flow rate pulsations, or a variety of other possible causes, die out very rapidly and the progressing flame front is more or less planar. In an unstable combustion process these perturbations grow rapidly and create a fingered combustion front which can bypass much of the potentially recoverable fossil fuel, thus making the process quite inefficient. Unstable in situ combustion is not always disadvantageous. For example, the fact that the
reverse combustion linking step in the in situ coal gasification process is unstable, is actually beneficial since it establishes a highly permeable link more rapidly with considerably reduced air requirements.

Several analyses have appeared in the literature which investigate the stability of in situ combustion processes. However, no one appears to have considered the stability of reverse wet combustion, nor has anyone presented a complete analysis for forward wet combustion. In this paper the authors investigate the linear stability of both forward and reverse in situ wet combustion for conditions such that there is sufficient water present to effect coincident steam and combustion fronts. This limiting case of wet combustion with coincident fronts is considered here for two reasons: (1) it will provide information on the effects of extremely high water injection on front stability; (2) the coincident-front problem cannot be derived as a special case of the double-front problem since in the former the front temperature is determined by the saturation temperature of steam, whereas in the latter it is determined by the air injection velocity.

RELATED STABILITY STUDIES

A recent review of the stability of moving combustion fronts in porous media is given by Armento and Miller. In this same paper these authors present an analysis for the linear stability of forward wet combustion with coincident steam and combustion fronts such that all the water is vaporized at the front. They found that forward wet combustion is only conditionally stable and is influenced by the ratio of the mobilities of the displacing and displaced fluids, heat transport from the combustion front, and expansion effects at the front. For low to moderate ratios of injected air to water, a low (favorable) mobility ratio was found to be stabilizing as would be expected from stability theory for displacement fronts. Expansion effects at the front due to gas generation from both combustion and water vaporization are destabilizing. The unstable modes were the long waves since the shorter waves have larger surface area for heat transfer. A surprising prediction was that at higher ratios of injected air to water, the effects of mobility ratio and expansion at the front were reversed. However, these results must be interpreted with some caution. The analysis developed here, which includes Armento and Miller's problem as a special limiting case, does not agree with their predictions. The reasons for these differences will be explained in the theoretical development.

The authors (Krantz and Gunn and Gunn and Krantz) recently analyzed the linear stability of in situ reverse dry combustion. At low to moderate air injection rates they found the process to be unstable with respect to long wavelength disturbances. Heat transfer from the front selectively stabilized the shorter waves. Low mobility ratios of displacing to displaced fluid were found to be destabilizing. However, expansion effects due to gas generation at the front had a stabilizing influence. These results suggested that large amounts of moisture in the coal seam could influence the reverse linking process by stabilizing shorter wavelength disturbances, thereby increasing the diameter of the combustion tubes and correspondingly the link-up time. These same expansion effects could possibly effect destabilization of the forward combustion step, thereby causing poor sweep efficiencies.

SOLUTION FOR UNPERTURBED COMBUSTION

A stability analysis seeks to determine whether small perturbations in the basic or initial state will grow or decay. In this section the solution for this basic state, which is assumed to be a planar combustion front, is developed.

Consider the idealization of the in situ wet combustion process shown in Figure 1. Air and water flow in the +\(n_x\) direction where \(n_x\) is a coordinate measured from the moving combustion front, which in this analysis is assumed to be coincident with the steam front at which the water is all or partially vaporized. The combustion front is assumed to travel at a constant velocity \(v^k\), such that \(v^k > 0\) for forward combustion and \(v^k < 0\) for reverse combustion. In forward combustion, region I is combusted and region II is uncombusted, whereas in reverse combustion the converse is true.

In both cases heat transfer from the front is assumed to occur only in region I since region II is heated by combustion gases to a constant temperature \(T_v\), the saturation temperature of steam at the prevailing pressure. The temperature of the surroundings far from the front is assumed to be \(T_o\).

It is convenient to cast the conservation equations in dimensionless form using the following dimensionless variables:

\[ T = \frac{T - T_o}{T_v - T_o}, \quad v = \frac{v}{v^*_g}, \quad \eta = \frac{\eta_v}{\eta_g}, \]

where

\[ \eta_g \equiv \frac{k_h (T_v - T_o)}{\rho_a F_o \Delta H_r v_y^g + \rho_w C_w \Delta H_w v_w^g (v_y^w - \phi_l S_w v^w)} \]

The length scale factor \(\eta_g\) is a measure of the penetration depth of conduction from the combustion front in region I. Its definition is suggested by balancing the net heat generation with the conduction terms in the energy balance at the combustion front. The gas and water injection velocities, \(v_y^g\) and \(v_w^g\), respectively, are assumed constant. In all cases, barred quantities denote time-average or unperturbed variables. Super-scripts asterisks denote dimensional quantities. The remaining parameters appearing above and elsewhere in this paper are defined in the nomenclature section.

The corresponding dimensionless form of the energy equation when written in a coordinate system convected at \(v^k\) is then given by

\[ \frac{d^2 T}{d \eta^2} - \frac{N Pe}{\eta_g} \frac{dT}{d \eta} = 0 \quad -\infty < \eta \leq 0 \]

where

\[ N Pe \equiv \frac{T_v - T_o}{k_h \rho_a F_o \Delta H_r v_y^g} \]

is a type of Pecllet number for this problem in which

\[ \Gamma \equiv \rho_g C_g v_y^g + \rho_w C_w (v_w^g - \phi_l S_w v^w) - \rho_s C_s v^k \]

This analysis assumes that the basic state is steady in a coordinate system convected at the combustion front velocity.

The dimensionless boundary conditions are given by

\[ T = 1 \quad \eta = 0 \]

\[ T = 0 \quad \eta = -\infty \]

Equation 6 demands that the combustion front temperature be the saturation temperature of steam at the prevailing pressure.
In order to specify this problem completely, four additional equations are required:

\[
\frac{dT}{dn} = 1 \quad \text{at} \quad n = 0 \tag{8}
\]

\[
(1 + F_{02} \rho_{g}) + \bar{T}_w \frac{\rho_{gw}}{\rho_{g}} (\bar{v}_{w1} - \phi_1 S_{w1} \bar{v}) = \frac{\rho_{g}}{\rho_{gw}} \bar{v}_{c} \tag{9}
\]

\[
(1 - \bar{T}_w) (\bar{v}_{w1} - \phi_1 S_{w1} \bar{v}) = (\bar{v}_{w2} - \phi_2 S_{w2} \bar{v}) \quad \text{at} \quad n = 0 \tag{10}
\]

\[
\frac{\rho_{s1}}{\rho_{s2}} + \frac{\rho_{g} F_{02} \rho_{gw}}{\rho_{gw}} = 1 \quad \text{at} \quad n = 0 \tag{11}
\]

where \( \bar{T}_w \) is the fraction of water vaporized, and \( \bar{v}_{c} \) and \( \bar{v}_{w2} \) are the combustion gas and water velocities, respectively, in region II. Equation 8 is the energy balance at the combustion front. Equations 9, 10, and 11 are the gas, water, and solid mass balances, respectively, at the front. These four equations imply that we have only two degrees of freedom in specifying the parameters \( \bar{v}_{g} \), \( \bar{v}_{w1} \), \( \bar{v}_{w2} \), \( \bar{v} \), and \( \bar{T}_w \). Armento and Miller consider the special case of \( \bar{T}_w = 1 \), that is, all the water is assumed to be vaporized at the front. This then implies that only one of the remaining five parameters need be specified to determine the others uniquely. Yet, in presenting their results Armento and Miller consider \( \bar{v}_{g} \) and \( \bar{v}_{w1} \) to be independently variable parameters. This overdetermination of the problem may account in part for the divergence between the results obtained here and those of Armento and Miller.

The solution to Equation 3 satisfying Equations 6 and 7 is given by

\[
\bar{T} = e^{N_{pe} \bar{n}} \quad \text{or} \quad \bar{T} = e^{N_{pe} T^*} \quad \text{or} \quad \bar{T} = e^{N_{pe} T^*} \tag{12}
\]

Equation 8 requires that

\[
N_{pe} = 1 \tag{13}
\]

These equations coupled with Equations 9, 10, and 11 uniquely determine the basic state of the system if the physical properties are known and any two of the parameters \( \bar{v}_{g} \), \( \bar{v}_{w1} \), \( \bar{v}_{w2} \), \( \bar{v} \), and \( \bar{T}_w \) are specified. This solution for the basic state can now be used to obtain a solution for the perturbed combustion problem.

**SOLUTION FOR PERTURBED COMBUSTION**

This analysis follows the formalism of linear stability theory wherein all variables are assumed to be of the form

\[
T = \bar{T} + T^*; \quad v = \bar{v}_1 + v_1^*; \quad p = \bar{p} + p^*; \quad f_w = \bar{f}_w + f_w^* \tag{14}
\]

where the superscript slashes denote infinitesimal perturbations and the subscript '1' implies any of the velocities including the front velocity. The above indicates another difference between this analysis and that of Armento and Miller. The latter investigators consider only the case \( \bar{T}_w = 1 \) and do not perturb \( f_w \). However, if the air or water injection velocities are perturbed then the energy balance, and gas and water mass balances at the front imply that \( f_w \) is perturbed as well. That is, although \( \bar{T}_w = 1 \) in the basic or unperturbed solution of Armento and Miller, it will depart from unity in the perturbed combustion solution. Since Armento and Miller have not perturbed \( f_w \), the results of the present analysis will not agree with their results for the special limiting case of \( \bar{T}_w = 1 \).

The appropriate form of the linearized energy equation in region I when transformed to a coordinate system translating at a constant velocity \( \bar{v} \) is

\[
\frac{3}{3n_1} \frac{\partial^2 \bar{T}^*}{\partial n_1^2} + \frac{3}{3n_2} \frac{\partial^2 \bar{T}^*}{\partial n_2^2} + \frac{3}{3n_3} \frac{\partial^2 \bar{T}^*}{\partial n_3^2} - (\rho g \phi g V_{g1}^* + \rho_{gw} V_{w1}^*) \quad \text{at} \quad n = 0 \tag{15}
\]

where

\[
N_{p} = \frac{\rho_{s1} \rho_{g} + \rho_{g} \rho_{gw} + \rho_{gw}}{\rho_{gw}} \tag{16}
\]

Armento and Miller ignore the last term in Equation 15, thus their analysis is restricted to describing the properties of neutrally and marginally stable disturbances whose temporal growth rate is negligibly small. This term will be retained here.

Equation 15 must satisfy the following boundary conditions:

\[
T^* + n_1 = 0 \quad \text{at} \quad n = 0 \tag{17}
\]

\[
T^* \rightarrow 0 \quad \text{as} \quad n \rightarrow - \infty \tag{18}
\]

These are just the perturbed forms of Equations 6 and 7.

The perturbed forms of Equations 8, 9, and 10 imply an interrelationship among the perturbed variables. Their form are then given by

\[
\frac{\partial \bar{T}^*}{\partial n} + \bar{n}_1 - \bar{n}_3 \frac{\bar{T}_w}{\rho_{gw}} - \bar{f}_w \bar{n}_w (\bar{v}_{w1} - \bar{f}_1 S_{w1} \bar{v}) - \bar{n}_w (\bar{v}_{w1} - \bar{f}_1 S_{w1} \bar{v}) \bar{f}_w^* = 0 \quad \text{at} \quad n = 0 \tag{19}
\]

where

\[
N_{a} = \frac{\rho_{a} \phi_{a} \rho_{gw} V_{gw1}^* + f_{w} \rho_{gw} V_{w1}^* - \phi_1 S_{w1} V_{w1}^*}{\rho_{gw} V_{gw1}^* + f_{w} \rho_{gw} V_{w1}^* - \phi_1 S_{w1} V_{w1}^*} \tag{20}
\]

\[
N_{w} = \frac{\rho_{a} \phi_{a} \rho_{gw} V_{gw1}^* + f_{w} \rho_{gw} V_{w1}^* - \phi_1 S_{w1} V_{w1}^*}{\rho_{gw} V_{gw1}^* + f_{w} \rho_{gw} V_{w1}^* - \phi_1 S_{w1} V_{w1}^*} \tag{21}
\]

\[
(1 + \phi_{a} \rho_{a} V_{gw1}^*) \bar{v}_{g1} + \bar{f}_w \rho_{gw} \bar{v}_{w1} - \phi_1 S_{w1} \bar{v}_{w1} \bar{v}_{w1}^* + \bar{f}_w \rho_{gw} \bar{v}_{w1} - \phi_1 S_{w1} \bar{v}_{w1} \bar{v}_{w1}^* \tag{22}
\]

\[
(1 - \bar{T}_w) (\bar{v}_{w1} - \phi_1 S_{w1} \bar{v}) \bar{f}_w^* = \frac{\rho_{g}}{\rho_{gw}} \bar{v}_{c} \quad \text{at} \quad n = 0 \tag{23}
\]

Equations 15 through 23 imply a coupling between the perturbed temperature and velocity profiles. The latter must satisfy Darcy's law and the incompressible continuity equation given in dimensionless form by

\[
\bar{v} \cdot \bar{n} = - \frac{\partial}{\partial n} \bar{p} = 0 \quad \text{for} \quad - \infty < n < 0 \tag{24}
\]

\[
\bar{v} \cdot \bar{v}_{w1} = - \frac{\partial}{\partial n} \bar{p} = 0 \quad \text{for} \quad 0 < n < \infty \tag{25}
\]

\[
\bar{v} \cdot \bar{v}_{c} = - \frac{\partial}{\partial n} \bar{p} = 0 \quad \text{for} \quad 0 < n < \infty \tag{26}
\]

\[
\bar{v} \cdot \bar{v}_{w2} = \frac{\bar{v}_{w2}^2}{\bar{v}_{c}^2} \quad \text{for} \quad 0 < n < \infty \tag{27}
\]
where $M_k$ is the ratio of the mobility of the displacing gas. The fact that in each region the gas and water flow are subject to the same pressure gradient has been used in simplifying the above equations. This same consideration implies the following two interrelationships between the perturbed velocities:

$$v_{w1} = \frac{v_w}{v_c} v'_g$$

and

$$v_{w2} = \frac{v_w}{v_c} v'_c$$

Since the pressure must be continuous at the combustion front we require that

$$p' - \eta' = p' - |\eta| = M_k v_c \eta'$$

The perturbed pressure in regions I and II can be obtained by integrating the perturbed form of Darcy's law in these regions.

The functional forms for $T'$, $v'_g$, $v'_{w1}$, $v'_c$, $v'_{w2}$, $p'$, and $\eta'$ must be of the following form in order to satisfy Equations 17, 19, and 22 through 30:

$$T' = \tau(\eta) f(y,z) e^{\beta t} \quad \text{for} \quad -\infty < \eta \leq 0$$

$$v'_g = \alpha g f(y,z) e^{\alpha \eta} e^{\beta t}$$

$$v'_{w1} = \alpha_{w1} f(y,z) e^{\alpha \eta} e^{\beta t}$$

$$v'_c = -\alpha c f(y,z) e^{\alpha \eta} e^{\beta t}$$

$$v'_{w2} = -\alpha_{w2} f(y,z) e^{\alpha \eta} e^{\beta t}$$

$$p' - M_k f(y,z) e^{\alpha \eta} e^{\beta t} \quad \text{for} \quad -\infty < \eta \leq 0$$

$$\eta' = \delta f(y,z) e^{\beta t}$$

The function $f(y,z)$ must satisfy the equation

$$\frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = -\alpha^2 f$$

This equation is satisfied by simple harmonic functions corresponding to having a dimensionless wave number $\alpha = 2\pi \alpha y / \lambda = (\alpha^2 + \alpha^2 z)^{1/2}$, where $\alpha_y$ and $\alpha_z$ are the wave numbers in the $y$- and $z$-directions, respectively. In this analysis the fossil fuel deposit has been assumed to be homogeneous. Therefore, one would anticipate three-dimensional disturbances with $\alpha_y = \alpha_z$; hence the perturbed combustion front will have channels with a circular cross-section of diameter $\sqrt{2\lambda}$.

Equations 19, 22, 23, 28, 29, and 30 constitute six equations relating six parameters. These are $f_w$ and the initial amplitudes of the other five perturbed variables $\phi_o$, $\phi_{w1}$, $\phi_c$, $\phi_{w2}$, and $\delta$. When five of these variables are expressed in terms of the sixth variable the resulting algebraic equation is found to be homogeneous in the latter variable. Thus, this constitutes an interrelationship among the physical and dynamical parameters characterizing the problem. In particular this relationship permits determining the temporal amplification factor $\beta$ as a function of the wave number $\alpha$ and the parameters characterizing the problem.

Equations 31 through 38 indicate that positive purely real values of $\beta$ imply unstable disturbances of wave number $\alpha$ which grow exponentially in time. Negative purely real values of $\beta$ imply stable disturbances which decay in time. Any complex roots for $\beta$ will occur in conjugate pairs such that the sum of these two particular solutions constitutes an oscillatory mode. These complex conjugate roots for $\beta$ correspond to disturbances which are oscillatory in time, while the real part of $\beta$ determines if unstable or stable oscillatory disturbances are produced. If any wave number $\alpha$ can be found for which $\beta > 0$, the process is said to be unstable. The locus of wave numbers satisfying the condition $\beta = 0$ defines the neutral stability curve corresponding to disturbances which neither grow nor decay.

**DISCUSSION OF THEORETICAL RESULTS**

The general expression for the temporal amplification factor, $\beta$, as a function of wave number, $\alpha$, is given by

$$\beta^3 + C_1 \alpha^2 \beta^2 + C_1 \alpha^4 \beta + C_4 \alpha^3 \beta + C_4 \alpha^5 = 0$$

where the six constants in this expression are defined in the nomenclature section. This solution was obtained by assuming that $v_{w1} >> \phi_1 v_w$ and $v_{w2} >> \phi_2 v_w$. Throughout this analysis we have invoked this assumption for the air and combustion gas velocities which are larger than the water velocities. Inclusion of some terms in the problem results in an extremely lengthy expression for $\beta$ and a maximum 3.0 percent change in the neutral stability results. Thus, only the simplified expression is presented here to better illustrate the dominant factors affecting the stability.

The resulting form for the neutral stability curve arising from Equation 40 is thus given by

$$\alpha_0 = -C_1 f_c = \frac{N_c - N_c + 2N_k (1 - N_k)}{4N_k (N_c - 1)(N_k - N_k)}$$

where

$$N_c = \frac{\rho_c \rho_s}{\rho_g \rho_v}$$

$$N_k = \frac{\rho_g \rho_s}{\rho_c (1 + F_0 M_c) + \rho_w \rho_v}$$

$$M_k = \frac{N_k}{N_c}$$

Note that $N_k$ is a modified mobility ratio which accounts for expansion effects at the front resulting from gas generation due to combustion and vaporization of water.

These simplifying assumptions imply that the neutral stability curve or locus of neutrally stable wave numbers is a function of only two dimensionless parameters. It is convenient then to present our results as a plot of $\alpha$ versus $N_k$ with $N_c$ as a parameter. Physical considerations along with Equation 13 imply that generally for reverse wet combustion ($\gamma < 0$) $-\infty < N_k < 0$ and $0 < N_k < 1$, whereas for forward wet combustion ($\gamma > 0$) $0 < N_k < 1$ and $0 < N_k < 1$.

The neutral stability curves for reverse wet combustion for several values of $N_k$ are shown in Figure 2. All wave numbers beneath a given neutral stability curve are unstable. Note that Equation 41 implies that $\alpha \rightarrow \alpha = N_k \rightarrow 0$ and $\alpha \rightarrow \alpha = N_k \rightarrow 1$. Equation 41 also indicates that $\alpha = 0$ only when $N_c = 0$ or $N_k = 1$. The first condition is physically inaccessible; the second condition cannot be ruled out completely although the porous matrix would have to exhibit very unusual relative permeability behavior for $N_k$ to have
a value of one or greater. Thus, reverse wet combustion is unstable for most physically realizable conditions.

It is instructive to consider the physical significance of these predictions. Reverse wet combustion is unstable for nearly all operating conditions because of the unfavorable mobility ratio. That is, $M_0 < 1$ since region I is uncombusted and contains more water, whereas region II is all or partially combusted and contains less water. Expansion effects at the front due to gas generation from combustion and vaporization are seen to be stabilizing effects since they tend to increase the group $N_k$. For a given value of $N_c$, increasing $N_k$ decreases the band of unstable wave numbers. Complete stabilization is effected when $N_k = 1$. However, this is physically unlikely as can be seen by recasting $N_k$ in an equivalent form given by

$$N_k = \frac{k_1 w_1}{k_2 k_w_2} \left(1 - \frac{T_w}{w_2}\right) = \frac{k_1 q}{k_2 k_c} (1 + F_{O_2} N_c) \tag{44}$$

Normally this group cannot equal unity even if $T_w = 0$, because in general $k_1 < k_2$ and $q < k_c$.

The neutral stability curves for forward wet combustion are shown in Figure 3. Equation 41 for this case indicates that the neutral stability curve has two branches. The branch extending from $\alpha = 0$ at $N_k = 1$ to $\alpha = N_c(2-N_c)/4(1-N_c)$ at $N_k \rightarrow \infty$ demarcates a region of stable short waves from a region of unstable oscillatory waves. The latter implies complex conjugate roots for $\beta$ such that $\beta_r > 0$; the sum of the two particular solutions constitutes a disturbance which is harmonic in time and whose amplitude increases exponentially in time. Although oscillatory modes have been reported for unstable preferential and thermal convection problems, this appears to be the first report of such modes for unstable in situ combustion fronts in porous media. The branch of the neutral stability curve extending from $\alpha = 0$ at $N_k = N_c(2-N_c)$ to $\alpha \rightarrow \infty$ at $N_k = N_c$ demarcates a region of stable nonoscillatory waves from a region of stable oscillatory waves. Thus, this latter branch is of particular practical interest. Since $N_k$ is generally greater than one for forward combustion, the forward wet combustion process will also be unstable for most operating conditions.

A physical interpretation of this process again points to the stabilizing influence of a favorable mobility ratio. However, note that for forward wet combustion, expansion effects at the front are a destabilizing effect and can effect destabilization of this process at values of $M_0 < 1$. Again, heat transfer from the front selectively stabilizes the shorter waves which have greater area for heat transfer.

The predictions of this analysis for forward wet combustion should include those of Armento and Miller1 for the special case of $\beta = 1$, that is, complete vaporization of water at the front. However, our results differ substantially from those of Armento and Miller. In particular, this analysis does not predict any regions for forward combustion for which favorable mobility ratios are found to be destabilizing and expansion effects to be stabilizing as did Armento and Miller. The possible reasons for this divergence in predictions between these two analyses have been cited in the text at the appropriate points in the theoretical development.

Linear stability theory is capable not only of predicting whether a process is unstable with respect to small perturbations, but also can predict some properties of these disturbances. Of particular interest is the wave number of the most amplified mode since this is the disturbance whose properties should determine the size of the combustion channels observed in practice. The predictions for the most highly amplified wave number at a reasonable value of $N_c$ for both reverse and forward combustion are shown in Figures 4 and 5, respectively.

The results of this analysis have rather interesting implications concerning the influence of significant amounts of water on the linked vertical well process for in situ coal gasification. The forward combustion gasification step in this process appears to be stable based on the field tests conducted at Hanna, Wyoming, by the Laramie Energy Technology Center. However, Figure 3 indicates that this forward gasification step could become unstable in very wet coal seams, thereby resulting in fingering and poor sweep efficiencies. On the other hand, Gunn and Krantz3 have shown that the reverse combustion linking step in this process is unstable. This is beneficial to the overall process since the resulting combustion channels permit rapid linking with a minimum amount of injected air. However, Figure 4 indicates that the combustion tube diameter becomes enlarged for very wet seams since the wave number of the most highly amplified wave decreases due to the stabilizing effects of expansion at the front. This could result in larger combustion tube diameters in the reverse combustion linking step and thus slower linking and increased air injection requirements.

CONCLUSIONS

1. Linear stability theory indicates that reverse wet combustion is unstable for nearly all physically realizable operating conditions for the special case of coincident steam and combustion fronts.

2. Forward wet combustion is conditionally stable. It can be destabilized with respect to oscillatory instabilities for unfavorable mobility ratios or when expansion effects due to gas generation from combustion or vaporization become significant.

NOMENCLATURE

$\alpha_c$, $\alpha_{p_1}$, $\alpha_u$ = amplitude factors defined by Eqs. 32 through 35
$C$ = heat capacity at constant pressure
$C_1$ = constant in Eq. 40, $\frac{N_c^3 N_c^2 (1 + N_k)^2}{4 N_k (N_k - N_c) (1 - N_c)^2}$
$C_{11}$ = constant in Eq. 40, $\frac{N_c^2 N_c^2 (1 + N_k) (3N_c - 3N_k - 1 - 4N_c N_k)}{4 N_k (N_k - N_c) (1 - N_c)^2}$
$C_{111}$ = constant in Eq. 40, $\frac{N_c^2 N_c^2}{(N_k - N_c) (1 - N_c)}$
$C_{1V}$ = constant in Eq. 40, $\frac{N_c^2 (1 - N_k) [N_c^2 (1 + N_k) - 2N_c (1 - N_c) - 2N_k]}{4 N_k (N_k - N_c) (1 - N_c)}$
$C_V$ = constant in Eq. 40, $\frac{N_c^2 N_c^2 (1 + N_k) + N_c (1 - N_k)^2}{2N_k (N_c - N_k) (1 - N_c)}$
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\[ C_{VI} = \text{constant in Eq. 40,} \quad \frac{N_C(N_{kC} + N_C - 2N_k)(1 - N_k)}{4N_k(N_C - N_k)(1 - N_C)} \]

\[ f(y,z) = \text{function defined by Eq. 39} \]

\[ f_w = \text{fraction of water vaporized} \]

\[ F_{O_2} = \text{weight fraction oxygen, region I} \]

\[ \Delta H_r = \text{heat of combustion per unit mass of oxygen consumed} \]

\[ \Delta H_w = \text{heat of vaporization of water} \]

\[ k = \text{permeability} \]

\[ k_c, k_g, k_w = \text{relative permeability of combustion gases, air, and water} \]

\[ k_h = \text{thermal conductivity} \]

\[ m_c = \text{mass of gas generated per unit mass of oxygen consumed} \]

\[ M_k = \text{mobility ratio,} \quad \frac{k_c \mu_c}{k_g \mu_g} \]

\[ N_A = \text{dimensionless group defined by Eq. 20} \]

\[ N_C = \text{dimensionless group defined by Eq. 42} \]

\[ N_k = \text{dimensionless group defined by Eq. 43} \]

\[ N_p = \text{dimensionless group defined by Eq. 16} \]

\[ N_{pe} = \text{Peclet number defined by Eq. 4} \]

\[ N_v = \text{inverse dimensionless combustion front velocity,} \quad N_v = 1 \frac{P_{s2} - P_{s1}}{P_{gF}O_2 m_c} \]

\[ N_w = \text{dimensionless group defined by Eq. 21} \]

\[ P = \text{pressure} \]

\[ S_w = \text{water saturation} \]

\[ t = \text{time} \]

\[ T = \text{temperature} \]

\[ T_{o} = \text{ambient temperature} \]

\[ T_v = \text{saturation temperature of steam} \]

\[ v = \text{combustion front velocity} \]

\[ v_c = \text{superficial combustion gas velocity in region II} \]

\[ v_g = \text{superficial gas velocity in region I} \]

\[ v_w = \text{superficial water velocity} \]

\[ x, y, z = \text{stationary coordinates} \]

\[ \alpha = \text{dimensionless wave number,} \quad 2\pi n_5/\lambda \]

\[ \alpha_m = \text{most highly amplified wave number} \]

\[ \alpha_n = \text{neutrally stable wave number} \]

\[ \beta = \text{dimensionless amplification factor} \]

\[ \Gamma = \text{dimensional quantity defined by Eq. 5} \]

\[ \delta = \text{amplitude factor defined by Eq. 38} \]

\[ \eta = \text{coordinate moving with velocity} \quad \overline{v} \]

\[ \eta_5 = \text{scale factor defined by Eq. 2} \]

\[ \lambda = \text{dimensional wavelength} \]

\[ \rho_c, \rho_g, \rho_s = \text{density of gas in region II, region I, and of solid, respectively} \]

\[ \mu = \text{viscosity} \]

\[ \tau = \text{temperature perturbation defined by Eq. 31} \]

\[ \phi = \text{porosity} \]

\[ c = \text{combustion gas region II} \]

\[ g = \text{gas in region I} \]

\[ s = \text{solid} \]

\[ w = \text{water} \]

\[ 1, 2 = \text{region I and II, respectively} \]

\[ \mu = \text{time-average or unperturbed quantity} \]

\[ \lambda = \text{dimensional quantity} \]

<table>
<thead>
<tr>
<th>References</th>
</tr>
</thead>
</table>
Fig. 1 - Idealized schematic representation for wet combustion processes

Fig. 2 - Neutral stability curves for reverse wet combustion

Fig. 3 - Neutral stability curves for forward wet combustion
**FIG. 4** - Neutral stability curve and locus of most highly amplified wave for reverse wet combustion

**FIG. 5** - Neutral stability curve and locus of most highly amplified wave for forward wet combustion
A THEORETICAL INVESTIGATION OF FORWARD COMBUSTION IN A ONE-DIMENSIONAL SYSTEM

by Vidya Bhushan Verma, Phillips Petroleum Company, Albert C. Reynolds and Gordon W. Thomas, University of Tulsa, Members SPE-AIME

ABSTRACT

This paper treats a theoretical investigation of forward combustion in a simulated laboratory combustion tube. To this end, a mathematical model is presented that incorporates the principal kinetic, thermal, thermodynamic and hydrodynamic aspects of the process. The model is similar to those presented by Gottfried and Crookston, except unlike the Gottfried model, we treat seven equations rather than five, and account for a mass balance on fuel, which incorporates both fuel laydown and burnoff. Furthermore, vaporization-condensation of both oil and water is included. Our model differs from the model of Crookston, insofar as we consider some of the oil vapor to participate in combustion. We further differ by employing a more rigid formulation of the problem at points where liquid phase disappearance occurs rather than using an arbitrary artifice to handle this situation. In addition, we use a sequential implicit solution technique which we found to be satisfactory.

With the exception of the gas phase, mass and heat balances are always maintained within less than 0.5%. We frequently see the same bounds on the gas balances, however, under some circumstances this bound is exceeded. This is expected since the gas saturation is calculated by difference, and thereby reflects the cumulative errors in the computation.

The model yields spatial distributions of pressure, temperature, oil, water, gas, and fuel saturations and/or concentrations. In addition, we can determine at any point and time the mole fractions of various components in the gas phase. Finally, it is possible to predict recovery factors, production rates, cumulative production, and instantaneous and cumulative WOR's and gas-liquid ratios.

In this paper, we present a detailed interpretation of the various computed profiles and identify some phenomena (e.g., the existence of an oil vapor plateau, peak "shaving" of the water vapor profiles, etc.) not previously identified in the literature. Comparisons of the computed results with laboratory experimental results are also presented.

INTRODUCTION

Thermal recovery methods are most promising for recovering low API gravity crudes, or oil from shales or tars. Basically the idea is to supply heat to the reservoir to lower the oil viscosity and enhance flow to the production wells. The heat can be generated at the surface, or in-place, and supplied either locally or globally through one or more injection wells. In most applications involving surface generation, steam is the heat medium. In-place generation involves the propagation of a combustion zone to globally affect the reservoir. The latter process is the concern of this work, in particular forward combustion, which we briefly describe below.

In forward combustion, the crude is ignited in the vicinity of an air injection well followed by continued air injection. Due to the heat, part of the oil ahead of the burning zone is vaporized, and displaced downstream by the incoming hot gases. A residue derived from the heavy ends of the oil is left behind to serve as fuel for the combustion front. This is referred to as coke. In addition, some of the oil vapors are undoubtedly consumed in the combustion reactions. The displaced oil vapor ahead of the front ultimately condenses to form an oil bank, which is driven to the production wells.

In the published literature, initial attempts were directed toward an analytical treatment of the heat flow mechanisms without a consideration of fluid flow effects. Heat transfer inside the system and losses to the surroundings were accounted for by conduction and/or convection. The combustion zone was simulated by incorporating a transient instantaneous heat source in the energy equations. All of these treatments were highly idealized insofar as the reaction kinetics of the process were not taken into account.

Thomas suggested the inclusion of finite reaction rates of fuel lay-down and fuel burn-off.
governed by Arrhenius-type reaction rate functions. He included mass balances of fuel and oxygen in a steady state model and concluded that the mechanisms of fuel lay-down and burn-off were essentially competitive. Chu\(^{33}\) incorporated the phenomena of vaporization and condensation and studied their effects on the temperature distribution and steam plateau behavior in a one-dimensional system, again assuming an instantaneous heat source. Gottfried\(^{27}\) presented a linear model, which included the features of heat transmission as well as three phase fluid flow, vaporization and condensation of water, and external heat loss by natural convection. The vaporization and condensation of crude oil and the fuel lay-down mechanism, however, were neglected. A number of numerical models have been presented in one and two dimensions incorporating single or multiphase fluid flow and heat transfer.\(^{15,13,20,21,32}\) However, the reaction kinetics and phase behavior were either largely simplified or neglected.

An elaborate two-dimensional model for thermal recovery processes has recently been presented by Crookston, et al.\(^{14}\) It accounts for the major phenomena involved in forward combustion including fuel lay-down and burn-off although solubility of gases in the liquids is neglected. Another two-dimensional model of forward combustion presented by Farouq Ali takes into account solubility effects, but the reaction kinetics and phase behavior are either largely simplified or neglected.

Recent experimental work\(^3\) indicates that some oil vapor is consumed in the combustion reactions. This has not been accounted for in previous mathematical models, nor is the significance of its effects known, although one may anticipate they play a minor role. Furthermore, our understanding of certain aspects of the problem is still incomplete. With this in view, our objectives were to develop a one-dimensional mathematical model that incorporates the major features of dry, forward combustion, including oil vapor burn-off, and to further investigate the thermo- and hydrodynamic phenomena of the process. At the same time, we present an alternative approach for handling liquid phase disappearance and numerically treating the differential equations than that proposed by Crookston, et al.\(^{14}\)

### Mathematical Formulation

Let us consider a one-dimensional homogeneous porous system analogous to a laboratory combustion tube in which oil, water, and their vapors and air exist in thermodynamic equilibrium under given initial conditions of temperature and pressure. Flow of fluids through the system is against a fixed back pressure at the downstream end. Ignition is treated as injection of heat at the inlet boundary.

The available heat is used to vaporize oil and water, to convert crude oil into coke, and to raise the temperature of the system. The amount of heat utilized for each of these occurrences is dependent upon the pressure, temperature, and the physical properties of the porous medium and its contained fluids. Heat is lost to the surroundings by convection, and within the system, heat is transported by conduction and convection.

### Reaction Kinetics

Although the process involves numerous reactions taking place simultaneously in the system, we restrict ourselves to four:

1) Fuel lay-down: Under the effect of heat, crude oil vaporizes and cracks, and leaves behind a bituminous-type material or coke residue. This serves as the main fuel for sustenance and propagation of the combustion front. This is a slightly endothermic reaction and can be represented in a simple form as:

\[
\text{Oil} + (\text{Heat}) \rightarrow X_1 (\text{fuel}) + X_2 (\text{light hydrocarbons})
\]

2) Fuel burn-off: In the presence of oxidant gas, \(O_2\), oxygen in air, the deposited fuel is consumed at a certain rate and reaction products in the form of carbon dioxide, carbon monoxide and water are obtained. This reaction is largely exothermic and can be stated as:

\[
\text{Fuel} + X_3 (O_2) \rightarrow X_4 (CO) + X_5 (CO_2) + X_6 (H_2O) + (\text{Heat})
\]

3) Oil burn-off: We postulate that part of the oil is also consumed by high temperature oxidation resulting in combustion products and generation of heat as follows:

\[
\text{Oil} + X_7 (O_2) \rightarrow X_8 (CO) + X_9 (CO_2) + X_{10} (H_2O) + (\text{Heat})
\]

4) Oil vapor burn-off: Oil vapor is also considered to participate in high temperature oxidation by the following exothermic reaction:

\[
\text{Oil vapor} + X_{11} (O_2) \rightarrow X_{12} (CO) + X_{13} (CO_2) + X_{14} (H_2O) + (\text{Heat})
\]

Note that the assumed burn-off reactions indicate that the combustion of coke, oil and its vapor is incomplete. This view is validated by the fact that some concentrations of carbon monoxide have always been detected in experimental flue gas analyses.\(^{3,4,10}\) The degree of completeness, which any or all of these reactions can attain, undoubtedly depends on the type of reactants, their kinetic characteristics, relative concentrations, and temperature, which are the main parameters of Arrhenius rate functions. Clays and metallic compounds appear to have a catalytic effect on these reactions.\(^{8,9,28}\)

To evaluate the stoichiometric coefficients \((X_i, i=1-11)\), in the above reactions we need to know the type of oil used, the nature of fuel or coke formed, and the composition of oil vapor. A given type of oil can be represented by an averaged hypothetical hydrocarbon compound, such that its physical properties (specific gravity, molecular weight, etc.) approximate those of the oil under consideration. A similar approach can be used for determining the composition of oil vapor and fuel.

The rates of all the above-mentioned reactions are governed by Arrhenius type reaction rate functions and can be described as follows:

\[
R_1 = Z_1 (\phi^0 S_0) e^{-\frac{E_1}{RT}}
\] (1)
Fig. 6 - Fuel concentration profiles

Fig. 7 - Distributions of oil vapor (mole fractions)

Fig. 8 - Water vapor distribution (mole fractions)

Fig. 9 - Oxygen concentration profiles (mole fractions)

Fig. 10 - Comparison of the computed and experimental results (run # 107)
1100 COMBUSTION TUBE RUN

DATE
JULY 21-22, 1976

Fig. 1 - Temperature distributions at various times

Fig. 2 - Temperature profiles in sand pack at every hour

Fig. 3 - Pressure profiles at various times

Fig. 4 - Oil saturation distributions

Fig. 5 - Water saturation distributions


TABLE 1 - INPUT DATA FOR THE BASE RUN

### Basic Data

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<td>h_a</td>
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### Initial and Boundary Data

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### Reaction Kinetic Data

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<td>X_11</td>
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Mole Fractions Of Gas Phase Components

As discussed earlier, the gas phase consists of oil vapor, water vapor, oxygen, and inert/flue gases. We shall look into the peculiarities of the distributions of first three of these components with a view to giving a physical interpretation of the numerical results obtained from our model. Theoretical as well as experimental data on these distributions are scarce. However, we attempt to make qualitative interpretations, wherever possible.

The oil vapor distributions at various times are presented in Fig. 7. Every profile is characterized by a sharp peak, which occurs at the leading edge of the combustion front with amplitudes depending on the temperature prevailing at that point. Since the leading edge of the combustion front may not at any moment coincide with a nodal point, the temperature is determined, the amplitudes in Fig. 7 vary from one moment in time to another.

The peak in every oil vapor profile coincides with the location of the corresponding primary oil bank. A zone of constant oil vapor concentration expanding with time travels ahead of this peak. Except for its relatively low magnitude, it resembles the steam plateau. We designate this zone as an oil vapor plateau. Due to the very low concentrations in the oil vapor plateau, its presence has not previously been detected in experimental studies. Nor has its existence been reported in theoretical studies. The existence of the oil vapor plateau is consistent with the phase equilibrium requirements of the system.

Although this phenomenon is distinct in the case of a crude oil represented by a single hydrocarbon, its effect could be smeared in a real situation, where crude oil is composed of a large number of hydrocarbons. In that case, oil vapor plateau the constituent hydrocarbons would be superimposed on one another.

The dynamics of the water vapor distribution is similarly connected with the propagation of the steam plateau as discussed under temperature profiles. The water vapor profiles presented in Fig. 8 are similar in nature to the oil vapor profiles, except here the concentrations are higher. Furthermore, as the concentration profiles develop we observe a flattening of the peaks. One would not expect the water vapor profiles to exhibit the same sharp behavior as in the oil profiles, since all the liquid water goes to steam as the trailing edge of the steam plateau passes through a cell.

The oxygen concentration profiles are characterized by a decline of concentration at the combustion front (Fig. 9). It is apparent that consumption of oxygen is incomplete. Undoubtedly, the amount of oxygen consumed will be dependent on the reaction parameters entering the burn-off reactions (activation energy, pre-exponential constant, order of reaction, concentrations of hydrocarbon reactants, and temperature). We observed that by changing these parameters, complete oxygen utilization can be attained. However, in several experimental studies, varying concentrations of oxygen have been detected in the exit gases. Therefore, incomplete utilization of oxygen seems to be realistic.

Comparing Experimental Results

The mathematical model was used to simulate the experimental data from a recent laboratory investigation of forward combustion. Although the quantitative comparisons were favorable as already discussed, it required a lot of matching effort to get close comparisons with the laboratory data because most of the kinetic data for the various reactions were not known with certainty. The match of temperature profiles in Fig. 10 illustrates the capability of our model to match laboratory combustion tube results. If the data are reliable, a good match can be obtained with little effort and meaningful predictions of future performance made.

Conclusions

A mathematical model incorporating the principal kinetic, thermal, thermodynamic, and hydrodynamic aspects of forward combustion in a one dimensional system has been developed. It includes the mechanisms of oil and water vaporization/condensation, fuel laydown and burnoff. Besides coke, the oil vapor and crude oil itself are also considered to participate in combustion. The disappearance of the liquid phase is treated by a reformulation procedure, which is rigorous from the mass conservation standpoint. The model has been tested to simulate experimental data, in particular, temperature profiles. Furthermore, physical interpretations of the computed profiles of temperature, pressure, saturations, and mole fractions have been presented.

1. Although the burnoff of crude oil and its vapor make a relatively small contribution to combustion compared to coke burnoff, for completeness, their inclusion in the combustion kinetics is desirable.
2. The existence and propagation of an oil vapor plateau is indicated. The concept of propagation of the oil vapor plateau gives better insight into the formation of the secondary oil bank.
3. In the water saturation profiles, a peak "shaving" effect has been identified which is caused by the conversion of water into steam and propagates in synchronization with the steam plateau.
4. The temperatures corresponding to the steam and oil vapor plateaus, which are the saturation temperatures of thermodynamic equilibrium between the liquid and vapor phases of water and oil, respectively, are governed by the partial pressure of the respective vapor, not by the total pressure of the system.
5. The temperature profiles computed theoretically have been matched with those obtained from a laboratory study with reasonable accuracy.

6. The fuel concentration distribution in the system follows the hypothesis of a two-step chain reaction confirming that the mechanisms of fuel laydown and burnoff are essentially competitive. It is illustrated by obtaining peaks in the fuel concentration profiles, which propagate along the leading edge of the combustion front.

7. Any investigation of the combustion process cannot be perfect, unless the kinetic data for various reactions involved in it are known with certainty from laboratory experiments. Therefore, a knowledge of these data is imperative for making meaningful predictions.

**NOMENCLATURE**

- \( a \) = external heat loss area per unit volume, \( \text{ft}^{-1} \)
- \( C \) = specific heat, \( \text{Btu/lb m}^2/\text{OF} \)
- \( E_i \) = activation energy, \( \text{Btu/lb m} \)
- \( H \) = external heat loss coefficient, \( \text{Btu/day ft}^2/\text{OF} \)
- \( k_i \) = reaction rate, \( \text{day}^{-1} \text{psia}^{-1} \)
- \( k_o \) = relative permeability
- \( K \) = thermal conductivity of rock, \( \text{Btu/day ft}^2/\text{OF} \)
- \( L \) = length of the system, ft
- \( N \) = number of nodes in the system
- \( P \) = pressure, psia
- \( Q \) = injection/production rate, \( \text{lb m/day} \)
- \( R \) = gas constant in reaction rate function, \( \text{psia ft}^3/\text{lb m} \)
- \( R_i \) = reaction rates, \( \text{lb m of the hydrocarbon reactant/day ft}^3 \)
- \( R(i) \) = composite reaction rates, \( \text{lb m/day ft}^3 \)
- \( S \) = saturation, fraction
- \( t \) = time, days
- \( T \) = temperature, \( ^\circ R \)
- \( V \) = flow velocity, \( \text{ft/day} \)
- \( x \) = spatial increment, ft
- \( X_i \) = stoichiometric coefficients in the reaction equations, \( \text{lb m} \)
- \( Z_i \) = mole fraction in the gas phase, \( \text{day}^{-1} \text{ft}^3 \)
- \( Z_i(v) \) = reaction rate constant in Arrhenius equation, \( \text{day}^{-1} \text{psia}^{-1} \)

**Greek Symbols**

- \( \phi \) = porosity, fraction
- \( \rho \) = molal density, \( \text{lb m/ft}^3 \)
- \( \mu \) = fluid viscosity, cp
- \( \lambda \) = heat of reaction, \( \text{Btu lb m} \)
- \( \nu \) = heat of vaporisation, \( \text{Btu lb m} \)
- \( \kappa \) = tolerance on a dependent variable
- \( \eta \) and \( \theta \) = reaction orders

**Subscripts**

- \( f \) = fuel
- \( g \) = gas
- \( H \) = heat injection
- \( i \) = general index or initial value
- \( j \) = injection
- \( k \) = index denoting a dependent variable
- \( m \) = maximum
- \( m \) = minimum
- \( o \) = oil
- \( v \) = oil vapor
- \( r \) = rock (relative permeability, when used with \( k \))
- \( s \) = surroundings
- \( T \) = total
- \( w \) = water
- \( w \) = water vapor
- \( 0 \) = initial

**Subscripts Used for Multiple Purposes**

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**Acknowledgments**

This work was performed at the University of Tulsa. We extend our thanks to the University for providing the computing facilities. Appreciation is also expressed to the Phillips Petroleum Company for providing assistance in preparing this manuscript.

**References**


Initially, temperature is the same everywhere in the system. At a given point, the temperature rises due to heat of conduction, convection, or reactions. When the temperature reaches the saturation value, further rise is checked since all the heat coming in or generated at this point is consumed in vaporising water and some oil. This constant temperature shows up as a steam plateau on the temperature profile. After all the liquid water disappears at that point, the temperature continues to rise again until the combustion temperature is reached.

The rate of propagation of the combustion front that we observe is higher than that noted in previous theoretical investigations. This rate is a function of the crude oil characteristics, the rates of air injection, and the kinetic parameters entering the reaction rate equations. Using the laboratory data of a recent study, the rates of combustion front propagation determined numerically and experimentally are found to be very close to 3.8 ft/day. This also falls within the range of rates observed by Martin, et al., for similar operating conditions.

Pressure Profiles

Pressure distribution in the system after every two hours is depicted in Fig. 3. Initially, the pressure is uniform everywhere. When injection commences, the pressure at the inlet end rises swiftly. Subsequently, the pressure gradient becomes nonuniform due to the creation of a liquid bank. The gradient is steeper across the liquid bank and is low in the areas of high gas saturation. For example, the oil and water banks at one hour are located between nodes three and ten (Figs. 4-5), which correspond to the steepest pressure gradients in this region in Fig. 3. We interpret this as follows.

Upon liquid build-up, the resistance to flow increases, resulting in higher pressure gradients. For the first three hours, the liquid banks keep increasing in size and therefore, the pressure keeps rising. Thereafter breakthrough occurs with a corresponding drop in pressure throughout the system. Every pressure profile is characterized by two slopes, one in the gas region and the other in the liquid region. With the combustion front approaching the outlet end, the pressure gradient becomes linear and pressures in the system approach a stabilized steady state distribution.

Saturation Profiles

Oil, water, and fuel saturation profiles are given in Fig. 4, 5, and 6, respectively. Each of these profiles exhibits several characteristic features, which are described below.

As soon as the injection of air is initiated, the oil and water banks start to build up near the inlet end. Typically, the oil saturation moves with two peaks, which we designate as the primary and secondary banks. The primary bank is characterized by an oil saturation peak less than or equal to the original oil saturation in the system. This peak is, however, seen to grow steadily with time. It moves immediately ahead of the combustion front and is separated from the secondary bank by an expanding trough-like depression in the oil saturation profile. This depression is caused by the steam zone stripping of the oil and its volatile components at high temperatures. Because the spatial extent of the steam zone increases with time, the trough stretches in length accordingly.

This fact is easily verified by superimposing the temperature profile (Fig. 1) or a water vapor concentration profile (Fig. 8) at a given time on the corresponding oil saturation profile. The condensation of oil vapor downstream results in the formation of the secondary bank, which has a saturation peak higher than the original oil saturation. The secondary bank also expands with time and breaks through at the outlet end quite early in the process. The breakthrough is recorded as a peak in the oil production rate. These results are in agreement with previous theoretical and experimental investigations.

Although the propagation of the water bank in Fig. 5 seems analogous to that of the secondary oil bank (Fig. 4), the mechanics involved in this case are somewhat different. Unlike the oil saturation profile, the water bank is not formed in the immediate vicinity of the combustion front. Instead, it coincides with the beginning of the steam plateau and extends far beyond it for any given time. This can also be seen by superimposing the temperature profile on a corresponding water saturation profile. At early times, the water bank has a sharp trailing edge, which gets smeared in the subsequent profiles, giving the impression of a partial "peak shaving." The length of the "shaved" portion exactly matches that of the steam plateau in the corresponding temperature profile, which implies that the "shaving" effect is brought about by the conversion of water into steam.

Gottfried displayed a similar character in his water saturation profiles. However, he interpreted this as the formation of an anomalous hump. In reality, the "shaving" effect gives a false appearance of a hump to the top portion of the "shaved" profile. There is no increase in water saturation, and so there is no anomaly in the form of a hump. Finally, it deserves to be mentioned that the mechanics of oil and water bank formation and their propagation will not only be governed by the pressure gradients and flow properties of the rock and its contained fluids, but also by the pressure-temperature-composition constraints, that dictate the phase behavior in the system.

The fuel concentration profiles (Fig. 6) are characterized by a peak, which always occurs at the leading edge of the combustion front as seen by superimposing temperature profiles on the fuel concentration profiles. This observation is consistent with experimental and theoretical investigations. Typically, the peak fuel concentration keeps increasing with time. This indicates that for the given set of conditions, the rate of fuel laydown is somewhat larger compared to fuel burn-off. Also, fuel accumulation ahead of the peak keeps increasing with time, since fuel is being generated, but only a small amount is consumed in the oxygen lean environment downstream. This results in a build-up of the fuel concentration. In the proximity of the combustion front, where the temperatures are high and a fresh supply of oxidant gas is readily available, fuel burn-off predominates and the fuel concentration quickly goes to zero.
SOLUTION PROCEDURE

We seek a solution of Equations (10)-(14), (16)-(19), and (22) subject to the given initial and boundary conditions for p, T, S_0, S_p, S_f, Y_1, Y_2, Y_3, and Y_4. Since equations (10)-(14) and (22) are nonlinear, we solve them by numerical methods. A completely implicit sequential finite difference procedure was adopted. To evaluate the nonlinear coefficients, the primary unknowns are extrapolated in time by a second order approximation, obtained from Taylor's series expansion.

We first obtain a pressure equation by combining the mass balances, Equations (10)-(14), and simplifying using the saturation and mole fraction constraints, Equations (16)-(17). The temperature equation is also modified by use of the conservation equations (See reference 30).

The nonlinear coefficients in the pressure and temperature equations are evaluated by extrapolating their arguments in time. The temperature and pressure equations are then solved sequentially by a tridiagonal algorithm. Using the new value of temperature and pressure, we update the nonlinear coefficients and solve these two equations again. These results are then employed to obtain S_0, S_p, S_f, Y_1, Y_2, and Y_4 from Equations (10), (11), (14), (18), (19), and (13), respectively. Finally, we compute S_p and Y_3 from Equations (16) and (17). This completes the first iteration. The nonlinear coefficients are again updated and the process repeated, except in subsequent iterations, the temperature and pressure equations are solved only once per iteration. Solving the temperature and pressure equations twice during the first iteration substantially increases the rate of convergence. At each step of each iteration we use the latest values of all variables. In this sense the procedure is analogous to a Gauss-Seidel procedure.

If we let u_k represent any of the dependent variables (where k = 1, 2, ..., N, N = total number of dependent variables), and k an iteration level, then convergence is achieved when \[ |u^{k+1} - u^k| \leq \varepsilon_p \]
for \( \varepsilon_p \) a specified tolerance. Alternatively, the iteration process can be terminated when a specified maximum number of iterations has been reached.

At the end of a time step, normalized heat and material balances are calculated to determine the validity of the results. We found in all runs that these were preserved with less than 0.5% error, with the exception of the gas which usually averaged approximately 5% error. Insofar as the gas saturations were calculated by difference, they naturally will reflect the total cumulative errors in the computation.

The time step size, \( \Delta t \), is changed in several stages between specified minimum and maximum values. Initially, \( \Delta t \) is calculated such that the maximum change in gas saturation (which changes fastest) in the first cell does not exceed 5% during the step. After a specified number of steps, \( \Delta t \) is increased by a given multiple. As we near a condition of liquid or solid phase disappearance, the time step size is changed as follows. If the extrapolated value of a liquid or coke saturation is negative, the extrapolation procedure is repeated with a smaller \( \Delta t \). This new value \( \Delta t \) is computed such that the extrapolated value of the phase saturation goes to zero. When phase disappearance occurs, the problem is reformulated and we solve for a different dependent variable. To avoid instabilities in the steps of this calculation, we use a time step size smaller that \( \Delta t \). Thereafter the \( \Delta t \)'s are gradually increased to the original value employed before phase disappearance.

Second order central difference approximations are utilized to discretize the spatial derivatives, whereas we use a backward difference in time. The interblock transmissibilities, fluid densities, viscosities, mole fractions, production, and reaction rates are dated implicitly in time. Upstream values of transmissibilities, densities and mole fractions are also used. After differencing, the pressure and temperature equations have the general algebraic form:

\[
\text{b}_i (u_{i-1} + a_i u_i + c_i u_{i+1} = d_i, \; i = 1, N) \]

When the boundary conditions are applied to this system of equations, we obtain a tridiagonal matrix problem of the form \( A \hat{u} = d \), where \( A \) is an \( N \)th order tridiagonal matrix, \( d \) a known vector, and \( \hat{u} \) is the solution vector of pressure or temperature.

DISCUSSION

The mathematical model yields, as functions of time, temperature and pressure distributions, oil, water, gas and fuel distributions, mole fractions of oil and water vapor, oxygen and inert gases, an oil recovery factor, the cumulative and instantaneous production of oil, water and gas, cumulative injection of air, and finally, cumulative and instantaneous water-oil and gas-liquid ratios. The data used for the base run are given in Table 1. We shall first discuss the characteristics of each of these variables as obtained from the base run. This is done with the help of profiles, which are obtained by taking a snapshot of the various distributions at different times.

Temperature Profiles

Figure 1 presents temperature profiles obtained from the base run. Each profile depicts the temperature distribution in the system at two hour intervals. The propagation of the combustion zone appears to occur at a uniform rate of approximately 3.8 ft/day. A steam plateau expanding with time travels ahead of the combustion front. Typically, the temperature of the steam plateau is equal to the saturation temperature, corresponding to the partial pressure of steam in the system. For the base run, the mole fraction of water vapor in the gas phase at 11 hours is 0.12 (Fig. 3). Therefore, the partial pressure of steam would be 31.2 psia. The saturation temperature corresponding to this pressure is 712°F, which is the temperature of the steam plateau in Fig. 1. The temperature profiles obtained from an experimental study display a striking similarity to those obtained from our mathematical model. The characteristic nature of these profiles can be best understood by the following physical interpretation.
Note, since the fuel occupies a part of the pore volume, we treat it as a "saturation", $S_f$, in Equation (19) and the pore volume available for the flowing fluids is reduced by that amount.

**Mass Transfer**

Mole fractions of oil and water vapor in the gas phase are calculated from vapor-liquid equilibrium considerations. Concentration of a liquid vapor in the gas phase at a point is a function of pressure and temperature at that point and can be computed by vapor pressure correlations for the given type of liquid. Correlations published in the literature are used for the particular type of oil and water considered, with suitable modifications; thus for a 30° API oil,

$$y_1 = \frac{1}{p} \exp \left( 13.1298 - \frac{10400 \cdot 0.62}{T - 149.377} \right) \quad (18)$$

and for water,

$$y_2 = \frac{1}{p} \left( \frac{2 - 5.5269 \cdot 10^6}{118} \right) \quad (19)$$

The above correlations cannot be used after the liquid phase disappears, since vapor-liquid equilibrium no longer holds. The problem is reformulated at that point, i.e., Equations (10) and (11) are solved for vapor mole fractions in the gas phase, $y_1$ and $y_2$, instead of liquid saturations, $S_0$ and $S_w$. Thus Equations (10) and (11) reduce to the following form:

$$-\frac{3}{\partial x} \left( v_g \rho_g y_1 \right) + R(1) = \frac{\partial}{\partial t} \left( \rho_g S_g y_1 \right) \quad (20)$$

and

$$-\frac{2}{\partial x} \left( v_g \rho_g y_2 \right) + R(2) = \frac{\partial}{\partial t} \left( \rho_g S_g y_2 \right) \quad (21)$$

This change causes a slight instability problem, which is resolved by decreasing the time step size in the vicinity of phase disappearance. In reality, the nature of a crude oil is more complex, and $k$-value routines should be used, with necessary modifications, for computing its phase behavior during combustion.

**Energy Balance**

Heat transport in the system occurs by both conduction and convection. Heat is generated by oxidation reactions and locally we can regard vaporization and condensation as heat consumption and generation processes, respectively. In an adiabatic system there are no heat losses. However, we incorporate a term to account for heat losses by natural convection when they are present. At the inlet end, heat is added to the system by a controlled heating element at a desired rate. Heat is also carried into the system by injected gases and carried out of the system by produced fluids. With these considerations in mind, applying the principle of energy conservation results in:

$$Q_{inj} (0, t) = \text{Const.} \quad (27)$$

$$\frac{\partial T}{\partial x} (0, t) = 0. \quad (28)$$

$$p (L + \frac{\Delta x}{2}, t) = p_e. \quad (29)$$

$$\frac{\partial T}{\partial x} (L, t) = 0. \quad (30)$$

The igniter (heat injection) is switched off in the first cell, when a given temperature is attained and is switched back on as soon as the temperature in this cell falls below a specified value.
Fuel burn-off:
\[ R_2 = Z_2 \left( \Phi \rho C_T^2 \right) \left( y_d \rho \right) \frac{E_2}{RT} \]  
(2)

Oil burn-off:
\[ R_3 = Z_3 \left( \Phi \rho C_o^3 \right) \left( y_o \rho \right) \frac{E_3}{RT} \]  
(3)

Oil vapor burn-off:
\[ R_4 = Z_4 \left( \Phi \rho C_o^4 \right) \left( y_o \rho \right) \frac{E_4}{RT} \]  
(4)

Thomas conjectured that the reactions of fuel lay-down and burn-off are competitive. Predominance of one or the other controls to a large extent the behavior of the combustion process. The quantitative role of each of these reactions in the composite process can be adequately understood only if the reaction rate parameters are known a priori. Estimates of reaction rate constants and activation energies \((Z_i, E_i, i = 1, 2, 3, 4)\) can be made experimentally for a given oil, coke, and oil vapor. These values are available only for the reactions of fuel lay-down and burn-off. The orders of reaction with respect to each reactant are assumed one, which is consistent with some published experimental results. There is a possibility they could be different for the individual reactions given in Equations (1)-(4), however, experimental data to establish this is lacking.

The net rates of consumption or generation of the main reactants can be written as follows for subsequent use in mass conservation equations:

- **Oil consumption rate:**
\[ R(1) = R_1 + R_2 + R_4 \]  
(5)

- **Water production rate:**
\[ R(2) = X_2 R_2 + X_8 R_3 + X_{11} R_4 \]  
(6)

- **Inert/flue gases production rate:**
\[ R(3) = X_2 R_1 + X_4 R_2 + X_7 R_3 + X_{10} R_4 \]  
(7)

- **Oxygen consumption rate:**
\[ R(4) = X_2 R_1 + X_6 R_3 + X_9 R_4 \]  
(8)

- **Fuel lay-down rate:**
\[ R(5) = X_1 R_1 - R_2 \]  
(9)

All the reaction rates are dependent upon the concentrations of the reactants involved, activation energies, reaction rate constants, orders of reaction, and temperature. Concentrations of the reactants can be evaluated by phase equilibrium correlations and mass balances on the desired phases. Similarly, temperature distributions can be determined by energy balance on the system.

**Mass Balances**

Let us superimpose a block-centered grid consisting of \(N\) blocks over a linear system of length \(L\) that is partitioned such that \(\Delta x/2 = x_1 < x_2 < x_3 \ldots < x_N = L - \Delta x/2\) where \(x_1 = (i-1/2)\Delta x, i = 1, 2, \ldots, N\). We seek mass conservation equations for each of the fluids flowing through this system (oil, water, and gas) and for the solid fuel. Gas is considered a mixture of oil vapor, water vapor, oxygen, and inert gases. The term inert gas is used to include all non-combustible gases, plus methane, which does not significantly contribute to the combustion process. The non-combustibles are nitrogen, carbon monoxide, carbon dioxide, and minor concentrations of helium and argon. The mole fractions of oil vapor and water vapor in the gas phase are determined from conditions of thermodynamic equilibrium and these are included in the conservation equations for their liquid counterparts. By applying the principle of mass conservation to various components of the system, the following equations are obtained:

- **Oil:**
\[ -\frac{3}{\Delta x} \left( v_0 y_0 + v_g y_1 \rho S_0 \right) - R(1) = \Phi \frac{3}{\Delta t} \left( \rho S_0 + \rho S_1 y_1 \right) \]  
(10)

- **Water:**
\[ -\frac{3}{\Delta x} \left( v_w y_w + v_g y_2 \rho S_w \right) + R(2) = \Phi \frac{3}{\Delta t} \left( \rho S_w + \rho S_2 y_2 \right) \]  
(11)

- **Inert Gases:**
\[ -\frac{3}{\Delta x} \left( v_g y_3 \rho S_g \right) + R(3) = \Phi \frac{3}{\Delta t} \left( \rho S_3 y_3 \right) \]  
(12)

- **Oxygen:**
\[ -\frac{3}{\Delta x} \left( v_g y_4 \rho S_g \right) - R(4) = \Phi \frac{3}{\Delta t} \left( \rho S_4 y_4 \right) \]  
(13)

- **Fuel:**
\[ R(5) = \Phi \frac{3}{\Delta t} \left( \rho S_o \right) \]  
(14)

The composite rates, \(R(i), i = 1-5\), are defined by Equations (5)-(9). Injection and/or production terms have not been written in the above equations, since these are included for the boundary cells with proper signs, when applicable. The flow velocities of the three fluid phases are given by Darcy's law
\[ v_i = -0.006328 \frac{\Delta x}{\mu_i} \cdot \frac{\partial P}{\partial x}, \quad i = o, w, g. \]  
(15)

We also have the constraints,
\[ S_o + S_w + S_g + S_f = 1 \]  
(16)

and
\[ \sum_{i=1}^{4} y_i = 1. \]  
(17)